PHY130: HW_11 Help

Solution or Explanation

(a) The work done on the gas in this constant pressure process is

$$W = -P(\Delta V) = -P\left(\frac{nRT_f}{P} - \frac{nRT_i}{P}\right) = -nR(T_f - T_i)$$

or $W = -(0.105 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(623 \text{ K} - 303 \text{ K}) = -279 \text{ J}$.

(b) The negative sign for work done on the gas indicates that the gas does positive work on its surroundings.

Solution or Explanation

(a) From the ideal gas law, $nR = PV_f/T_f = PV_i/T_i$. With pressure constant this gives

$$T_f = T_i\left(\frac{V_f}{V_i}\right) = (276 \text{ K})(2) = 5.53 \times 10^2 \text{ K}.$$

(b) The work done on the gas is

$$W = -P(\Delta V) = -(PV_f - PV_i) = -nR(T_f - T_i) = -nR(2T_i - T_i)$$
$$= -(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})[1(276 \text{ K})] = -2.30 \times 10^3 \text{ J} = -2.30 \text{ kJ}.$$

Solution or Explanation

(a) The work done on the gas is negative because it expands to a larger volume so that $W = -152$ J. Apply the first law of thermodynamics to find the change in internal energy, ΔU.

$$\begin{aligned}\Delta U &= Q + W \\ &= (915 \text{ J}) - (152 \text{ J}) \\ &= 763 \text{ J}\end{aligned}$$

(b) Use the relation $\Delta U = nC_v\Delta T$ to find the change in temperature. Here, $n = 4.00$ moles and $C_v = \frac{3}{2}R$ (for neon, a monatomic gas) to find the following.

$$\Delta U = nC_v\Delta T = \frac{3}{2}nR\Delta T \rightarrow \Delta T = \frac{\Delta U}{\frac{3}{2}nR}$$

$$\Delta T = \frac{763 \text{ J}}{\frac{3}{2}(4.00 \text{ mol})\left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)} = 15.3 \text{ K}$$

Solution or Explanation

(a) We treat the sprinter's body as a thermodynamic system and apply the first law of thermodynamics, $\Delta U = Q + W$. Then, with $\Delta U = -8.1 \times 10^5$ J and $W = -4.4 \times 10^5$ J (negative because the sprinter does work on the environment), the energy absorbed as heat is

$$Q = \Delta U - W = -8.1 \times 10^5 \text{ J} - (-4.4 \times 10^5 \text{ J}) = -3.70 \times 10^5 \text{ J}.$$

(b) The negative sign in the answer to part (a) means that energy is transferred from the sprinter to the environment by heat.

Solution or Explanation

The maximum possible efficiency for a heat engine operating between reservoirs with absolute temperatures of $T_c = 27° + 273 = 300$ K and $T_h = 381° + 273 = 654$ K is the Carnot efficiency

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{300 \text{ K}}{654 \text{ K}} = 0.541 \text{ or } 54.1\%.$$

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

(a) The coefficient of performance of a heat pump is COP = $|Q_h|/W$, where $|Q_h|$ is the thermal energy delivered to the warm space and W is the work input required to operate the heat pump. Therefore,

$$|Q_h| = W \cdot \text{COP} = (P \cdot \Delta T) \cdot \text{COP} = \left[\left(6.94 \times 10^3 \ \frac{\text{J}}{\text{s}}\right)(24 \text{ h})\left(\frac{3{,}600 \text{ s}}{1 \text{ h}}\right)\right]3.70 = 2.22 \times 10^9 \text{ J}.$$

(b) The energy extracted from the cold space (outside air) is

$$|Q_c| = |Q_h| - W = |Q_h| - \frac{|Q_h|}{\text{COP}} = |Q_h|\left(1 - \frac{1}{\text{COP}}\right)$$

or

$$|Q_c| = (2.22 \times 10^9 \text{ J})\left(1 - \frac{1}{3.70}\right) = 1.62 \times 10^9 \text{ J}.$$

Solution or Explanation

The potential energy lost by the log is transferred away by heat, so the energy transferred from the log to the reservoir is $\Delta Q_r = mgh$. The change in entropy of the reservoir (universe) is then

$$\Delta S = \frac{Q_r}{T} = \frac{mgh}{T} = \frac{(75.0 \text{ kg})(9.80 \text{ m/s}^2)(26.0 \text{ m})}{298 \text{ K}} = 64.1 \text{ J/K}.$$

Solution or Explanation

The maximum rate at which the body can dissipate waste heat by sweating is

$$\frac{Q}{\Delta t} = \left(\frac{m}{\Delta t}\right)L_v = \left(1.6 \ \frac{\text{kg}}{\text{h}}\right)\left(2430 \times 10^3 \ \frac{\text{J}}{\text{kg}}\right)\left(\frac{1 \text{ h}}{3{,}600 \text{ s}}\right) = 1080 \text{ W}.$$

If this represents 82% of the maximum sustainable metabolic rate [i.e., $Q/\Delta t = 0.82\,(\Delta U/\Delta t)_{max}$], then that maximum rate is

$$\left(\frac{\Delta U}{\Delta t}\right)_{max} = \frac{Q/\Delta t}{0.82} = \frac{1080 \text{ W}}{0.82} = 1320 \text{ W}.$$